Duality symmetry and Kerker conditions

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We unveil the relationship between two anomalous scattering processes known as Kerker conditions and the duality symmetry of Maxwell equations. We generalize these conditions and show that they can be applied to any particle with cylindrical symmetry, not only to spherical particles as the original Kerker conditions were derived for. We also explain the role of the optical helicity in these scattering processes. Our results find applications in the field of metamaterials, where new materials with directional scattering are being explored.

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In 1958, the United States made a significant advance on stealth technology by changing the geometry of aircrafts and significantly reducing the scattering of radar frequencies from them. Since then, lots of developments have followed up [1]. More recently, with the advent of the modern field of metamaterials, stealth technology has boosted its reach. Nowadays, bodies can be camouflaged not only by changing their geometrical properties, but also by modifying their optical properties. In this research direction, Kerker *et al.* [2] made an important contribution in 1983. In their paper, the scattering from a single sphere was considered and two anomalous scattering conditions were found. The first of them, known as the first Kerker condition, proved that the back scattered radiation from a sphere can be reduced to zero when the relative electric permittivity and magnetic permeability are equal, i.e., $\varepsilon_r = \mu_r$. In contrast, the second Kerker condition, only valid for small spheres, proves that the forward scattering of a particle can be dimmed to zero when $a_1 = -b_1$, where a_1 and b_1 are the first order electric and magnetic Mie coefficients, respectively [3]. This field of research has received a lot of attention recently, as researchers seek to independently control both the electric and magnetic resonances of different structures [4,5]. Finally, and long after it was predicted, Geffrin and co-workers [6] have been able to measure these two conditions in the microwave regime. In addition, very recently the first Kerker condition has also been measured in the dipolar approximation by two experimental groups. Person *et al.* [7] have measured the first Kerker condition using GaAs nanoparticles, whereas Fu *et al.* [8] have been able to switch from the first Kerker condition to the second one by using silicon nanoparticles. Nonetheless, the measurement of the Kerker conditions in the optical regime for arbitrary large particles still remains an unsolved problem.

In this Letter, we unveil the relationship between the Kerker conditions and the electromagnetic (EM) duality symmetry. We will also show that these anomalous scattering conditions are present in a larger variety of particles, spheres being a particular case. The first Kerker condition (or zero-backscattering) is an example of a dual system with cylindrical symmetry, and the second one (or zero-forward scattering) is an example of what we call an anti-dual system. These new findings enable us to generalize the Kerker conditions and extend their use to macroscopic optical systems, regardless of their size.

The proof of these statements is intimately related with optical helicity. Helicity is a property of fundamental particles. It is defined as the projection of the angular momentum (AM) onto the normalized linear momentum, i.e., $\Lambda = \mathbf{J} \cdot \mathbf{P}/|\mathbf{P}|$ whose eigenvalues are $p = \pm 1$ [9]. It commutes with all the components of the linear and AM operators, $\mathbf{P}$ and $\mathbf{J}$. Helicity is a well defined quantity for EM fields. For monochromatic fields, it can be written as the differential operator $\Lambda = (\nabla\times)/|k|$ [9]. A very insightful representation of the helicity is obtained in the Fourier space, i.e., when a beam is decomposed into plane waves. There, helicity measures the polarization handedness in all the plane waves. If all the plane waves have the same circular polarization with respect to its momentum vector, then the beam will have a well defined helicity, otherwise it will not. Last but not least, the helicity operator is known to be the generator of the generalized duality transformations in the source-free Maxwell equations [10]. Hence, when a system preserves the helicity of an EM field upon scattering, we will say that this system is dual. In contrast, if a system flips the helicity of a field from p to $-p$ upon scattering, we will say that this system is anti-dual.

Now, as we want to find the relationship between the Kerker conditions and the EM duality symmetry, we will center our attention on the interaction between spherical particles and light beams with well-defined helicity. Hence, we will use a modified version of the generalized Lorenz–Mie theory (GLMT) to solve the scattering problem. The GLMT solves the interaction between an incident EM field propagating in a lossless, homogeneous, isotropic medium and a homogeneous isotropic sphere [11]. The problem is described with three EM fields, the incident ($\mathbf{E}^i$), the scattered ($\mathbf{E}^{\text{sca}}$), and the interior ($\mathbf{E}^{\text{int}}$) one. The temporal dependence is considered to be $\exp(-i\omega t)$. Taking advantage of the rotational symmetry of the problem, the three fields are decomposed into the multipolar basis $\mathbf{A}_{jm_z}^{(y)}$ and then the boundary conditions are applied. The multipolar modes are a complete basis of solutions of Maxwell equations and are the most appropriate modes to describe EM problems with spherical symmetry. They are eigenvectors of the total AM operator J^2 and one of its projections such as J_z with respective values j and m_z [11]. Furthermore, in GLMT it is also required that they are eigenvectors of the parity operator Π. This gives, as a result, two different families of multipoles, magnetic and electric $(y) = (m, e)$, with two different parity values. In this way, these functions match the symmetries of the sphere and therefore they can be assumed to scatter independently. This is, however, not the only possible choice for the multipolar basis. Instead of splitting the multipolar modes into two families of modes with different values of the parity, it is possible to split them into two families of modes with well defined helicity. These modes with well defined helicity are the ones that we will use in our modified version of GLMT, as we want to relate the Kerker conditions and duality symmetry. We will denote these new multipoles as $\{\mathbf{A}^{+}_{jm_z}, \mathbf{A}^{-}_{jm_z}\}$. They can be written as a superposition of the previous ones $\{\mathbf{A}^{(m)}_{jm_z}, \mathbf{A}^{(e)}_{jm_z}\}$:

$$\mathbf{A}^{+}_{jm_z} = \frac{\mathbf{A}^{(m)}_{jm_z} + i\mathbf{A}^{(e)}_{jm_z}}{\sqrt{2}} \qquad \mathbf{A}^{-}_{jm_z} = \frac{\mathbf{A}^{(m)}_{jm_z} - i\mathbf{A}^{(e)}_{jm_z}}{\sqrt{2}}. \tag{1}$$

In order to obtain these expressions one must take into account that $\Lambda\mathbf{A}^{(m)}_{jm_z} = i\mathbf{A}^{(e)}_{jm_z}$ and $\Lambda\mathbf{A}^{(e)}_{jm_z} = -i\mathbf{A}^{(m)}_{jm_z}$ [11]. Consequently, it can be checked that $\Lambda\mathbf{A}^{\pm}_{jm_z} = \pm\mathbf{A}^{\pm}_{jm_z}$. Hence, a general decomposition of a beam with a well defined helicity will be of the form $\mathbf{E}^i = \sum_{j,m_z}\alpha^p_{jm_z}\mathbf{A}^p_{jm_z}$. Now, if we excite a sphere with a beam of this sort, the fields can be expressed as:

$$\begin{aligned}\mathbf{E}^i &= \sum_{j,m_z}\alpha^p_{jm_z}\mathbf{A}^p_{jm_z}\\ \mathbf{E}^{\mathrm{sca}} &= \sum_{j,m_z}\alpha^p_{jm_z}\left[\frac{a_j + b_j}{\sqrt{2}}\mathbf{A}^p_{jm_z} + \frac{a_j - b_j}{\sqrt{2}}\mathbf{A}^{-p}_{jm_z}\right]\\ \mathbf{E}^{\mathrm{int}} &= \sum_{j,m_z}\alpha^p_{jm_z}\left[\frac{d_j + c_j}{\sqrt{2}}\mathbf{A}^p_{jm_z} + \frac{d_j - c_j}{\sqrt{2}}\mathbf{A}^{-p}_{jm_z}\right],\end{aligned} \tag{2}$$

where $\{a_j, b_j, c_j, d_j\}$ are the Mie coefficients [12], $p = \pm 1$ is the helicity of the beam, and $\alpha^p_{jm_z}$ are the amplitudes that determine the multipolar content of the incident beam. In particular, if the incident beam is cylindrically symmetric, e.g., a focused circularly polarized Gaussian beam, the field must have a well defined J_z and the expression for the multipolar amplitudes takes the simplified form of $\alpha^p_{jm_z} = f(j, p)\delta_{m_z,M}$, where $f(j, p)$ is a general function of j and p, the Dirac delta forces all amplitudes with $m_z \neq M$ to be zero and therefore, all amplitudes with $j < M$ are also zero [11]. The Mie coefficients only depend on the size parameter of the problem $(x = 2\pi a/\lambda)$, the relative permeability (μ_r), and permittivity (ε_r) of the sphere with respect to the surrounding medium. Here, a is the radius of the particle and λ the wavelength in free space.

Now, it is very important to note that in Eq. (2), the scattered and interior fields do not generally preserve the helicity of the incident field (see Figs. 1(a) and 1(b)). Indeed, although a beam with a well defined helicity such as $\mathbf{E}^i$ impinges on an isotropic and homogeneous sphere, in general the scattered and the interior fields ($\mathbf{E}^{\mathrm{sca}}$ and $\mathbf{E}^{\mathrm{int}}$) will not be eigenvectors of the helicity. This is a consequence of the fact that the two pairs of Mie coefficients $\{a_j, b_j\}$ and $\{c_j, d_j\}$ are not generally equal, and therefore the amplitudes of the helicity components opposite to the incident one will be different from zero. However, as it was proven by Kerker *et al.* [2], $a_j(x) = b_j(x)\forall j, x$ when $\varepsilon_r = \mu_r$. This is the so-called first Kerker condition, and it implies zero-backscattering from the sphere in consideration. Nonetheless, observing Eq. (2) it can be seen that the first Kerker condition also implies that the sphere in consideration will preserve the helicity of the incident field $\mathbf{E}^i$ upon scattering and therefore be dual. Indeed, for the scattered field, the amplitudes of the components with opposite helicity to the incident one are given by $\alpha^p_{j,m_z}(a_j - b_j)/\sqrt{2}$. Hence, when $a_j = b_j$ they are all zero, granting the scattered field with the same helicity as the incident one. This feature has been overlooked in the past, and it will enable us to generalize the first Kerker condition. The first Kerker condition is a particular case of a more general condition that restores the EM duality symmetry in material media [10]. In general,

$$\frac{\varepsilon_i}{\mu_i} = \mathrm{const} \Leftrightarrow \Lambda \text{ conservation}, \tag{3}$$

for any medium made of an arbitrary number of isotropic and homogeneous submedia i, ε_i and μ_i being its electric permittivity and magnetic permeability. That is, when an arbitrary light beam impinges on a medium such that the electric and magnetic fields behave symmetrically, i.e., condition (3) is met, the helicity of this beam is not changed regardless of the geometry of this medium.

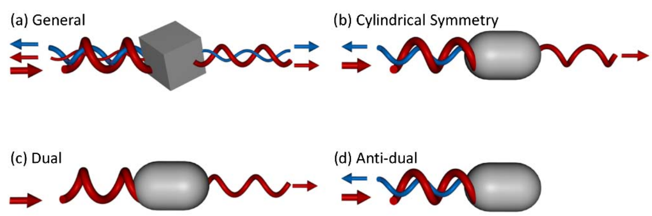

Fig. 1. Sketch of the relations between Kerker conditions and cylindrical and duality symmetry. Large red helix pointing the scatterer represents an incident plane wave with well defined helicity. The other helices represent scattered plane waves. Small red helix has the same helicity as the incoming wave, while the small blue helix has the opposite helicity. (a) General scattering process with neither cylindrical nor dual symmetries is shown. In general, both back and forward scattering have contributions from both helicities. (b) Scattering from a cylindrically symmetric object. In this case, symmetry imposes that forward scattering is of the same helicity than the incident field, and backward scattering of opposite helicity. (c) Dual and cylindrically symmetric system. Only forward scattering is allowed with the same helicity. (d) Anti-dual system with cylindrical symmetry. Only back scattering is allowed with flipped helicity.

Now, let us show that the zero-backscattering condition does not only apply to spherical scatterers, but also to any system with cylindrical symmetry. We will use a dual scatterer and impose that it must also be cylindrically symmetric around the z axis. Then, besides helicity, the J_z of the incident beam must also be preserved. Under these conditions, a plane wave incident along the symmetry axis will not backscatter. The proof comes from the definition of helicity $\Lambda = \mathbf{J} \cdot \mathbf{P}/|\mathbf{P}|$. Imagine that a plane wave is directed in the z axis (its linear momentum is $\mathbf{P} = P_z \hat{\mathbf{z}}$) and has a well defined helicity $\Lambda = J_z \cdot P_z/|P_z| = J_z = 1$. A plane wave with $\mathbf{P} = -P_z \hat{\mathbf{z}}$ must necessarily have $\Lambda = -J_z$. If the system preserves J_z and Λ, such a plane wave can never exist. Thus, it is very natural to generalize the first Kerker condition from a sphere such that $\varepsilon_r = \mu_r$ to a dual system with cylindrical symmetry, i.e., a system that preserves both the helicity and z component of the AM (see Fig. 1(c)), as both entities have zero backscattering.

We can also generalize the second Kerker condition, which predicts zero forward scattering for dipolar magnetic particles. In order to do this, let us first recall what we define as an “anti-dual” system. It is a scatterer whose scattered light always has an opposite helicity with respect to an incoming field with well defined helicity. That is, if $\mathbf{E}^i = \sum \alpha_{jm_z} \mathbf{A}^p_{jm_z}$, then $\mathbf{E}^{\text{sca}} = \sum \beta_{jm_z} \mathbf{A}^{-p}_{jm_z}$. We can then find the conditions for an anti-dual sphere again with the help of Eq. (2). For the scattered field, the amplitudes of the components with the same helicity as the incident field are given by $\alpha^p_{j,m_z}(a_j + b_j)/\sqrt{2}$. Thus, a necessary and sufficient condition for a sphere to be anti-dual is that $a_j = -b_j \forall j$. If we now express this condition in the dipolar approximation, i.e., we only consider the term with $j = 1$, we obtain $a_1 = -b_1$. As we mentioned in the introduction, this is the so-called second Kerker condition.

With this perspective we now understand that the zero-forward scattering condition for spheres (second Kerker condition) is again a particular case of a more general system, a system which is cylindrically symmetric and anti-dual. Once again, only considering the symmetries of such a system, it is possible to derive that the forward scattering must be zero. We use again, as an incident field, a plane wave travelling in the positive z axis, whose helicity is $\Lambda = J_z \cdot P_z/|P_z| = 1$. Now, as the system is anti-dual, the helicity in the forward direction will have to be $\Lambda = -1$. However, J_z cannot change since the system is cylindrically symmetric, and P_z will not change because we are considering the plane wave in the forward direction. Then, this plane wave cannot exist (see Fig. 1(d)). However, finding an exact anti-dual sphere is challenging. Some authors have noticed that dielectric, isotropic, and homogeneous spheres cannot behave as anti-dual materials in the strict sense, as they would contradict the optical theorem [13,14]. A possible anti-dual sphere could be made of a material with gain though, as it can be checked that it would not contradict the optical theorem.

To conclude, we have unveiled the relationship between the Kerker conditions and the EM duality symmetry. We have generalized them and we have also proved that the zero-backward and zero-forward scattering conditions are naturally fulfilled by dual and anti-dual systems when they are also cylindrically symmetric. We expect that these results can play an important role in the next generation of dielectric-based metamaterials [7,8], as the requirements to achieve these two null-scattering conditions have been relaxed.

References

1. P. W. Merlin, Am. Inst. Aeronaut. Astronaut. Pap. **2009**, 1522 (2009).
2. M. Kerker, D.-S. Wang, and C. L. Giles, J. Opt. Soc. Am. **73**, 765 (1983).
3. B. García-Cámara, F. González, F. Moreno, and J. M. Saiz, J. Opt. Soc. Am. A **25**, 2875 (2008).
4. A. García-Etxarri, R. Gómez-Medina, L. S. Froufe-Pérez, C. López, L. Chantada, F. Scheffold, J. Aizpurua, M. Nieto-Vesperinas, and J. J. Sáenz, Opt. Express **19**, 4815 (2011).
5. A. Kuznetsov, A. Miroshnichenko, Y. H. Fu, J. Zhang, and B. Luk’yanchuk, Science Reports **2**, 492 (2012).
6. J. Geffrin, B. García-Cámara, R. Gómez-Medina, P. Albella, L. Froufe-Pérez, C. Eyraud, A. Litman, R. Vaillon, F. González, M. Nieto-Vesperinas, J. J. Sáenz, and F. Moreno, Nat. Commun. **3**, 1171 (2012).
7. S. Person, M. Jain, Z. Lapin, J. J. Sáenz, G. Wicks, and L. Novotny, Nano Lett. **13**, 1806 (2013).
8. Y. H. Fu, A. I. Kuznetsov, A. E. Miroshnichenko, Y. F. Yu, and B. Luk’yanchuk, Nat. Commun. **4**, 1527 (2013).
9. I. Fernandez-Corbaton, X. Zambrana-Puyalto, and G. Molina-Terriza, Phys. Rev. A **86**, 042103 (2012).
10. I. Fernandez-Corbaton, X. Zambrana-Puyalto, N. Tischler, A. Minovich, X. Vidal, M. L. Juan, and G. Molina-Terriza, arXiv:1206.0868 (2012).
11. X. Zambrana-Puyalto, X. Vidal, and G. Molina-Terriza, Opt. Express **20**, 24536 (2012).
12. C. F. Bohren and D. R. Huffman, *Absorption and Scattering of Light by Small Particles* (Wiley, 1983).
13. M. Nieto-Vesperinas, R. Gomez-Medina, and J. J. Saenz, J. Opt. Soc. Am. A **28**, 54 (2011).
14. A. Alu and N. Engheta, J. Nanophoton. **4**, 041590 (2010).